Social Choice Theory Problem Set

1. Consider majority voting over pairwise alternatives subject to agenda manipulation. Use the following voting rules and preference profiles. There are three propositions to choose among, A, B, and C. There are three voters, 1, 2, 3.

The notation > indicates strict preference.
Rules: There is a chairman who sets the agenda, the order of voting. He announces two propositions to choose between; the winner of that vote faces a runoff against the remaining alternative.
Profile I:
Voter 1: A > B > C
Voter 2: B > C > A
Voter 3: C > A > B
Profile II:
Voter 1: A > B > C
Voter 2: B > C, B > A, (C vs. A preference is unspecified)
Voter 3: C > B > A
Claim: Under Profile I the chair can arrange that any one of the three propositions be the winner by the chair's choice of the order of voting. Under
Profile II, the choice is independent of the order of the agenda.

(a) Demonstrate the claim.

(b) Show that Profile II fulfills the concept of "single peaked preferences" and that Profile I does not.

2. Reference material for this question is reproduced below.

(a) How does the result in question 1 illustrate Black's Single Peaked Preferences Theorem (reproduced below)?

(b) The Arrow Possibility Theorem says that it is not generally possible to find a social choice mechanism fulfilling (1) non-dictatorship, (2) unrestricted domain, (3) independence of irrelevant alternatives, (4) Pareto principle. But simple majority voting on pairwise alternatives seems to do well in Profile II in question 1, and indeed, for all single peaked preference arrays. Is this a counterexample to the Arrow Possibility Theorem? If not, which of the four properties is (are) not fulfilled. Explain.

Reference material for question 2: Suppose all propositions to be decided by group decision-making can be linearly ordered, left to right. All voters agree on the left to right ordering. They disagree on their choices. Everyone has his favorite point; the favorite point differs among voters. For each voter, as we move to the left of his favorite, his utility goes down; as we move to the right of his favorite, his utility goes down. If this description is fulfilled, voter preferences are said to be "single-peaked" and we have

Theorem (Duncan Black): If preferences are single-peaked, then majority voting on pairwise alternatives yields transitive group decisions.

Problems 3, 4, 5, 6 deal with the Arrow Possibility Theorem. Recall that an Arrow Social Welfare Function is a group decision-making mechanism. In order to be successful the decisions must fulfill transitivity of the resulting choice ordering. Eliminating any one of the four restrictions (non-dictatorship, independence, Pareto principle, unrestricted domain) on an Arrow Social Welfare function allows us to find a successful Arrow SWF. That is, for any three of the four conditions, there is an Arrow Social Welfare Function that can fulfill those three and result in transitive group preferences. As usual, assume a finite number of voters, and at least three alternatives and voters. Demonstrate this result by finding a suitable Arrow SWF for each of the four sets of three conditions described below. You may find the following group choice mechanisms useful: Majority voting with single-peaked preferences; dictatorship; imposition of a preference ordering by constitution; weighted voting.

3. Find an Arrow SWF fulfilling non-dictatorship, independence, Pareto principle, but not unrestricted domain. Explain why your choice generates transitive preferences.

4. Find an Arrow SWF fulfilling non-dictatorship, independence, unrestricted domain, but not Pareto principle. Explain why your choice generates transitive preferences.

5. Find an Arrow SWF fulfilling non-dictatorship, Pareto principle, unrestricted domain, but not independence of irrelevant alternatives. Explain why your choice generates transitive preferences.

6. Find an Arrow SWF fulfilling independence, Pareto principle, unrestricted domain, but not non-dictatorship. Explain why your choice generates transitive preferences.